

CLAIMS:

1. An apparatus for computing the sum of a divisor $D_1=g.c.d.$ $((a_1(x)), (y-b_1(x)))$ and a divisor $D_2=g.c.d.$ $((a_2(x)), (y-b_2(x)))$ on Jacobian of a hyperelliptic curve $y^2+y=f(x)$ defined over $GF(2^n)$, said apparatus comprising:

a storage for storing $a_1(x)$, $a_2(x)$, $b_1(x)$ and $b_2(x)$; and
means for calculating $q(x)=\{s_1(x)(b_1(x)+b_2(x))\} \bmod a_2(x)$
or $q(x)=\{s_2(x)(b_1(x)+b_2(x))\} \bmod a_1(x)$ by using $s_1(x)$ or $s_2(x)$
in $s_1(x)a_1(x)+s_2(x)a_2(x)=1$ in case of $GCD(a_1(x), a_2(x))=1$ where
GCD denotes a greatest common divisor of two polynomials.

2. An apparatus for calculating $a'(x)$ and $b'(x)$ of a reduced divisor $D'=g.c.d.$ $((a'(x)), (y-b'(x)))$ which is a linearly equivalent to D_1+D_2 for a divisor $D_1=g.c.d.$ $((a_1(x)), (y-b_1(x)))$ and a divisor $D_2=g.c.d.$ $((a_2(x)), (y-b_2(x)))$ on Jacobian of a hyperelliptic curve $y^2+y=f(x)$ defined over $GF(2^n)$, said apparatus comprising:

means for calculating $q(x)=s_1(x)(b_1(x)+b_2(x)) \bmod a_2(x)$
by using $s_1(x)$ in $s_1(x)a_1(x)+s_2(x)a_2(x)=1$ in case of

$\text{GCD}(a_1(x), a_2(x))=1$ where GCD denotes a greatest common divisor of two polynomials;

means for calculating

$\alpha(x)=Q(q^2(x)a_1(x), a_2(x))+Q(f(x), a_1(x)a_2(x))$ which is rendered a monic polynomial where $Q(A, B)$ is a quotient of A/B ;

means for calculating $\beta(x)=(q(x)a_1(x)+b_1(x)+1) \bmod \alpha(x)$;

means for calculating $a'(x)=Q(f(x)+\beta^2(x), \alpha(x))$; and

means for calculating $b'(x)=(\beta(x)+1) \bmod a'(x)$.

3. An apparatus for computing the sum of a divisor $D_1=g.c.d.$

$((a_1(x)), (y-b_1(x)))$ on Jacobian of a hyperelliptic curve $y^2+y=f(x)$ defined over $GF(2^n)$, said apparatus comprising:

a storage for storing $a_1(x)$, and $b_1(x)$; and

means for calculating $q(x)=Q(b_1^2(x)+f(x) \bmod a_1^2(x), a_1(x))$

where $Q(A, B)$ is a quotient of A/B .

4. An apparatus for calculating $a'(x)$ and $b'(x)$ of a reduced divisor $D'=g.c.d. ((a'(x)), (y-b'(x)))$ which is a linearly equivalent to D_1+D_1 for a divisor $D_1=g.c.d. ((a_1(x)), (y-b_1(x)))$ on Jacobian of a hyperelliptic curve

$y^2+y=f(x)$ defined over $GF(2^n)$, said apparatus comprising:

means for calculating $q(x)=Q(b_1^2(x)+f(x) \bmod a_1^2(x), a_1(x))$

where $Q(A,B)$ is a quotient of A/B ;

means for calculating $\alpha(x)=q^2(x)+Q(f(x), a_1^2(x))$ which is rendered a monic polynomial;

means for calculating $\beta(x)=(b_1^2(x)+f(x) \bmod a_1^2(x)+1) \bmod \alpha(x)$;

means for calculating $a'(x)=Q(f(x)+\beta^2(x), \alpha(x))$; and

means for calculating $b'(x)=(\beta(x)+1) \bmod a'(x)$.

5. A method for calculating $a'(x)$ and $b'(x)$ of a reduced divisor $D'=g.c.d.((a'(x)), (y-b'(x)))$ which is a linearly equivalent to D_1+D_2 for a divisor $D_1=g.c.d.((a_1(x)), (y-b_1(x)))$ and a divisor $D_2=g.c.d.((a_2(x)), (y-b_2(x)))$ on Jacobian of a hyperelliptic curve $y^2+y=f(x)$ defined over $GF(2^n)$, said method comprising the steps of:

calculating and storing in a storage
 $q(x)=\{s_1(x)(b_1(x)+b_2(x))\} \bmod a_2(x)$ by using $s_1(x)$ in
 $s_1(x)a_1(x)+s_2(x)a_2(x)=1$ in case of $GCD(a_1(x), a_2(x))=1$ where GCD denotes a greatest common divisor of two polynomials;

calculating and storing in a storage
 $\alpha(x) = Q(q^2(x)a_1(x), a_2(x)) + Q(f(x), a_1(x)a_2(x))$ which is rendered a monic polynomial where $Q(A, B)$ is a quotient of A/B ;

calculating and storing in a storage $\beta(x) = (q(x)a_1(x) + b_1(x) + 1) \bmod \alpha(x)$;
 calculating and storing in a storage
 $a'(x) = Q(f(x) + \beta^2(x), \alpha(x))$; and
 calculating and storing in a storage $b'(x) = (\beta(x) + 1) \bmod a'(x)$.

6. A method for calculating $a'(x)$ and $b'(x)$ of a reduced divisor $D' = g.c.d.((a'(x)), (y - b'(x)))$ which is a linearly equivalent to $D_1 + D_1$ for a divisor $D_1 = g.c.d.((a_1(x)), (y - b_1(x)))$ on Jacobian of a hyperelliptic curve $y^2 + y = f(x)$ defined over $GF(2^n)$, said method comprising the steps of:

calculating and storing in a storage $q(x) = Q(b_1^2(x) + f(x) \bmod a_1^2(x), a_1)$ where $Q(A, B)$ is a quotient of A/B ;

calculating and storing in a storage $\alpha(x) = q^2(x) + Q(f(x), a_1^2(x))$ which is rendered a monic polynomial;

calculating and storing in a storage $\beta(x) = (b_1^2(x) + f(x)) \mod a_1^2(x) + 1 \mod \alpha(x)$;

calculating and storing in a storage $a'(x) = Q(f(x) + \beta^2(x)) \alpha(x)$; and

calculating and storing in a storage $b'(x) = (\beta(x) + 1) \mod a'(x)$.

7. A method for computing the sum of a divisor $D_1 = g.c.d. ((a_1(x)), (y - b_1(x)))$ and a divisor $D_2 = g.c.d. ((a_2(x)), (y - b_2(x)))$ on Jacobian of a hyperelliptic curve $y^2 + y = f(x)$ defined over $GF(2^n)$, said method comprising the steps of:

storing $a_1(x)$, $a_2(x)$, $b_1(x)$ and $b_2(x)$; and

calculating and storing in a storage $q(x) = s_1(x)(b_1(x) + b_2(x)) \mod a_2(x)$ or $q(x) = \{s_2(x)(b_1(x) + b_2(x))\} \mod a_1(x)$ by using $s_1(x)$ or $s_2(x)$ in $s_1(x)a_1(x) + s_2(x)a_2(x) = 1$ in case of $GCD(a_1(x), a_2(x)) = 1$.

8. A method for computing the sum of a divisor $D_1 = g.c.d. ((a_1(x)), (y - b_1(x)))$ on Jacobian of a hyperelliptic curve $y^2 + y = f(x)$ defined over $GF(2^n)$, said method comprising the

steps of:

storing $a_1(x)$, and $b_1(x)$; and

calculating and storing in a storage $q(x) = Q(b_1^2(x) + f(x))$
 $\text{mod } a_1^2(x), a_1(x)$) where $Q(A,B)$ is a quotient of A/B .